

On the $n=4$ Supersymmetry for the FRW ModelA. Pashnev^{a*}, J.J. Rosales^{b†}, V.I. Tkach^{b‡}, and M. Tsulaia^{a§}*^aBogoliubov Laboratory of Theoretical Physics, JINR
Dubna, 141980, Russia**^bInstituto de Fisica, Universidad de Guanajuato
C.Postal 66318, 05315-970 Leon, Gto. Mexico***Abstract**

In this work we have constructed the $n = 4$ extended local conformal time supersymmetry for the Friedmann-Robertson-Walker (FRW) cosmological models. This is based on the superfield construction of the action, which is invariant under worldline local $n = 4$ supersymmetry with $SU(2)_{local} \otimes SU(2)_{global}$ internal symmetry. It is shown that the supersymmetric action has the form of the localized (or superconformal) version of the action for $n = 4$ supersymmetric quantum mechanics. This superfield procedure provides a well defined scheme for including supermatter.

PACS number: 04.65.+e, 98.80.Hw

*e-mail: pashnev@thsun1.jinr.ru

†e-mail: juan@ifug3.ugto.mx

‡e-mail: vladimir@ifug3.ugto.mx

§e-mail: tsulaia@thsun1.jinr.ru

I. INTRODUCTION

The supersymmetric quantum mechanics (SUSY QM), being introduced first in [1] for the $n = 2$ case turns out to be convenient tool for investigating problems of the supersymmetric field theories, since it provides the simple and at the same time quite adequate understanding of various phenomena arising in the relativistic theories. For instance, one-dimensional [2] and multidimensional [3,4] $n = 4$ (SUSY QM) can be associated with $n = 1$, $D = 4$ supersymmetric field theories (including supergravity) subject to an appropriate dimensional reduction down to $D = 1$. Applications of supersymmetric quantum mechanics to the theory of black holes and to the other problems have been reviewed in [5,6]. This underlines the importance of studying simplified models in order to understand the key features of more complicated four dimensional problems. Along this research line, in recent years a systematic way to construct models of local supersymmetric quantum cosmology has been proposed [7,8]. This was performed by introducing a superfield formulation. This is because superfields defined on superspace allow all the component fields in a supermultiplet to be manipulated simultaneously in a manner which automatically preserves supersymmetry. This approach has the advantage of being simpler than the proposed models based on full supergravity [9,10], and by means of this local symmetry procedure it gives the corresponding fermionic partners in a direct manner. Using the superfield formalism the $n = 2$ local supersymmetric quantum mechanics for the cosmological models was constructed in [7,8], and a normalizable wavefunction of the Universe for the FRW cosmological model was obtained in [11]. Although these models do not attempt to describe the real world, they keep many of the features occurring in four dimensional space-time, which could really be studied in the quantum version of simplified models.

The most physically interesting case is provided by the $n = 4$ local supersymmetry, since it can be applied to the description of the systems resulting from the “realistic” $n = 1$, $D = 4$ supergravity subject to an appropriate dimensional reduction down to $D = 1$.

In the present report we propose a description of the FRW model based on the superfield construction of an action invariant under $n = 4$ worldline local supersymmetry with the $SU(2)_{local} \otimes SU(2)_{global}$ internal symmetry. Due to the invariance of the action we obtain the constraints, which form a closed superalgebra of the $n = 4$ supersymmetric quantum mechanics.

II. $N = 4$ SUPERCONFORMAL TRANSFORMATIONS

We begin by considering the homogeneous and isotropic metric defined by

$$ds^2 = -N^2(t)dt^2 + R^2(t)d\Omega_3^2, \quad (1)$$

where $d\Omega_3^2$ is the spatial FRW standard metric over three-space. The lapse function $N(t)$ and the scale factor $R(t)$ depend on the time parameter only. To construct the superfield action in the worldline superspace $(t, \theta^a, \bar{\theta}_a)$ (with t being a time parameter, and θ^a and $\bar{\theta}_a = (\theta^a)^*$, ($a = 1, 2$ is an $SU(2)$ index) being two complex Grassmann coordinates) one introduces a real “matter” superfield $\mathcal{R}(t, \theta^a, \bar{\theta}_a)$ and a worldline superreibein $\mathcal{N}(t, \theta^a, \bar{\theta}_a)$ which have

the following properties with respect to the $SU(2)$ $n = 4$ superconformal transformations of the worldline superspace [12] **

$$\delta t = \Lambda - \frac{1}{2}\theta^a D_a \Lambda - \frac{1}{2}\bar{\theta}_a \bar{D}^a \Lambda, \quad \delta \theta^a = i\bar{D}^a \Lambda, \quad \delta \bar{\theta}_a = iD_a \Lambda, \quad (2)$$

$$\delta \mathbb{R} = -\Lambda \dot{\mathbb{R}} + \dot{\Lambda} \mathbb{R} - i(D_a \Lambda)(\bar{D}^a \mathbb{R}) - i(\bar{D}^a \Lambda)(D_a \mathbb{R}), \quad (3)$$

$$\delta \mathbb{N} = -\Lambda \dot{\mathbb{N}} - \dot{\Lambda} \mathbb{N} - i(D_a \Lambda)(\bar{D}^a \mathbb{N}) - i(\bar{D}^a \Lambda)(D_a \mathbb{N}), \quad (4)$$

where dot denotes the time derivative $\frac{d}{dt}$. The transformation law (3) for the superfield \mathbb{R} shows that this superfield is a vector superfield in the one-dimensional $n = 4$ superspace, while the superfield $\mathbb{N}\mathbb{R}$ is a scalar [12].

The superfields \mathbb{R} and \mathbb{N} obey the quadratic constraints

$$[D_a, \bar{D}^a] \mathbb{R} = -4m, \quad D^a D_a \mathbb{R} = 0, \quad \bar{D}_a \bar{D}^a \mathbb{R} = 0, \quad (5)$$

and

$$[D_a, \bar{D}^a] \frac{1}{\mathbb{N}} = 0, \quad D^a D_a \frac{1}{\mathbb{N}} = 0, \quad \bar{D}_a \bar{D}^a \frac{1}{\mathbb{N}} = 0, \quad (6)$$

where

$$D_a = \frac{\partial}{\partial \theta^a} - \frac{i}{2} \bar{\theta}_a \frac{\partial}{\partial t}, \quad \bar{D}^a = \frac{\partial}{\partial \bar{\theta}_a} - \frac{i}{2} \theta^a \frac{\partial}{\partial t}, \quad (7)$$

are the supercovariant derivatives, and m is an arbitrary constant. The infinitesimal superfield

$$\begin{aligned} \Lambda(t, \theta, \bar{\theta}) &= a(t) + \theta^a \bar{\alpha}_a(t) - \bar{\theta}_a \alpha^a(t) + \theta^a (\sigma^i)_a{}^b \bar{\theta}_b b_i(t) \\ &\quad + \frac{i}{4} (\theta \theta) \bar{\theta}_a \dot{\bar{\alpha}}^a(t) - \frac{i}{4} (\bar{\theta} \bar{\theta}) \theta^a \dot{\alpha}_a(t) + \frac{1}{16} (\theta \theta) (\bar{\theta} \bar{\theta}) \ddot{a}(t), \end{aligned} \quad (8)$$

contains the parameters of local time reparametrizations $a(t)$, local supertranslations $\alpha(t)$, $\bar{\alpha}(t)$. $b_i(t)$ being a local $SU(2)$ parameter of the worldline superspace. The constraint (5) can be explicitly solved, the solution being described by the superfield

$$\begin{aligned} \mathbb{R}(t, \theta, \bar{\theta}) &= R'(t) + \theta^a \bar{\lambda}'_a(t) - \bar{\theta}_a \lambda'^a(t) + \theta^a (\sigma_i)_a{}^b \bar{\theta}_b F'_i(t) + m(\bar{\theta} \bar{\theta}) \\ &\quad + \frac{i}{4} (\theta \theta) \bar{\theta}_a \dot{\bar{\lambda}}'^a(t) - \frac{i}{4} (\bar{\theta} \bar{\theta}) \theta^a \dot{\lambda}'_a(t) + \frac{1}{16} (\theta \theta) (\bar{\theta} \bar{\theta}) \ddot{R}'(t). \end{aligned} \quad (9)$$

This superfield contains one bosonic field R' , which is associated with the scale factor $R(t)$ (see Eq. (12)) of the FRW model, the Grassmann-odd (they are four) fermionic fields $\lambda^a(t)$

**Our conventions for spinors are as follows: $\theta_a = \theta^b \varepsilon_{ba}$, $\theta^a = \varepsilon^{ab} \theta_b$, $\bar{\theta}_a = \bar{\theta}^b \varepsilon_{ba}$, $\bar{\theta}^a = \varepsilon^{ab} \bar{\theta}_b$, $\bar{\theta}_a = (\theta^a)^*$, $\bar{\theta}^a = -(\theta_a)^*$, $(\theta \theta) \equiv \theta^a \theta_a = -2\theta^1 \theta^2$, $(\bar{\theta} \bar{\theta}) \equiv \bar{\theta}_a \bar{\theta}^a = (\theta \theta)^*$, $(\theta \theta) \equiv \bar{\theta}_a \theta^a$, $\varepsilon^{12} = -\varepsilon^{21} = 1$, $\varepsilon_{12} = 1$.

and $\bar{\lambda}_a(t)$ are their superpartners being spin degrees of freedom, and $F_a^b = (\sigma^i)_a^b F_i$ are three auxiliary fields where $(\sigma^i)_a^b$ ($i=1,2,3$) are the ordinary Pauli matrices.

The constraint (6) being described by the superfield

$$\begin{aligned} \frac{1}{N}(t, \theta, \bar{\theta}) = & \frac{1}{N(t)} + \theta^a \bar{\psi}'_a(t) - \bar{\theta}_a \psi'^a(t) + \theta^a (\sigma^i)_a^b \bar{\theta}_b V'_i(t) \\ & + \frac{i}{4}(\theta\theta)\bar{\theta}_a \dot{\bar{\psi}}'^a(t) - \frac{i}{4}(\bar{\theta}\bar{\theta})\theta^a \dot{\psi}'_a + \frac{1}{16}(\theta\theta)(\bar{\theta}\bar{\theta})\frac{d^2}{dt^2}\frac{1}{N(t)}. \end{aligned} \quad (10)$$

The superfield N describes an $n = 4$ worldline supergravity multiplet consisting of the einbein “graviton” $N(t)$, two complex “gravitinos” $\psi'^a(t)$ and $\bar{\psi}'_a(t)$, and the $SU(2)$ gauge field $V'_i(t)$. The components of N play the role of Lagrange multipliers. Their presence implies that the dynamics of the model is subject to constraints.

The $n = 4$ superfield action for the FRW model invariant under the $n = 4$ superconformal symmetry has the following form [12]

$$S = \frac{8}{\varkappa^2} \int dt d^2\theta d^2\bar{\theta} N^2 \mathbb{R}^3, \quad (11)$$

where $\varkappa^2 = 8\pi G_N$, G_N is the Newtonian constant of gravity. So, integrating (11) over the Grassmann coordinates $\theta, \bar{\theta}$ and making the following redefinition of the component fields

$$\begin{aligned} \psi &= N^{\frac{3}{2}}\psi', \quad V_i = 2N(V'_i + N(\psi'\sigma_i\bar{\psi}')), \quad \lambda = \sqrt{N}(\lambda' - R\psi'), \\ F_i &= 2\sqrt{N}\{F'_i - RV'_i + \frac{\sqrt{N}}{2}(\psi'\sigma_i\bar{\lambda}) + \frac{\sqrt{N}}{2}(\lambda\sigma_i\bar{\psi}')\} \quad R = NR', \end{aligned} \quad (12)$$

one obtains the component action

$$\begin{aligned} S = \frac{1}{\varkappa^2} \int \left\{ -\frac{3R(DR)^2}{N} - 6iR[(\bar{\lambda}D\lambda) + (\lambda D\bar{\lambda})] - 3RF_i F^i + 6\sqrt{N}(\lambda\sigma^i\bar{\lambda})F_i \right. \\ \left. - 3[(\bar{\lambda}\bar{\lambda})(\lambda\psi) + (\bar{\lambda}\bar{\psi})(\lambda\lambda)] + 12m^2NR + 12mN(\bar{\lambda}\lambda) - 12mR[(\bar{\lambda}\psi) + (\bar{\psi}\lambda)] \right\} dt, \end{aligned} \quad (13)$$

where $DR = \dot{R} - i[(\bar{\psi}\lambda) - (\bar{\lambda}\psi)]$ is the supercovariant derivative, $D\lambda = \dot{\lambda} + \frac{i}{2}(\sigma^i)\lambda V_i$ and its conjugate are the $SU(2)$ covariant derivatives. Upon solving for the equations of motion of the auxiliary fields F_i , substituting the solution back into (13) and making the redefinitions $\lambda \rightarrow \varkappa(6R)^{-1/2}\lambda$, $\bar{\lambda} \rightarrow \varkappa(6R)^{-1/2}\bar{\lambda}$, and putting $4m^2 = k$ (where $k = 1, 0, -1$ stands for spherical, plane or hyperspherical three space), we have then

$$\begin{aligned} S = \int \left\{ -\frac{3R(DR)^2}{\varkappa^2 N} - i[(\bar{\lambda}D\lambda) + (\lambda D\bar{\lambda})] + \frac{3k}{\varkappa^2}NR + \frac{N\sqrt{k}}{R}(\bar{\lambda}\lambda) \right. \\ \left. - \frac{\sqrt{k}\sqrt{6R}}{\varkappa}[(\bar{\lambda}\psi) + (\bar{\psi}\lambda)] - \frac{\varkappa}{\sqrt{24}R^{3/2}}[(\bar{\lambda}\bar{\lambda})(\lambda\psi) + (\bar{\lambda}\bar{\psi})(\lambda\lambda)] - \frac{N\varkappa^2}{8R^3}(\bar{\lambda}\bar{\lambda})(\lambda\lambda) \right\} dt. \end{aligned} \quad (14)$$

The expression (13), as well as the expressions (17)-(19) for the Hamiltonian and Supercharges, can be considered also for the hyperspherical geometry at the value $4m^2 = -1$ when they lost their hermiticity properties. At least formally this should not lead to contradictions in the physical sector which is singled out by the equations (32) in the quantum case.

Performing Legendre transformations one arrives at the first order form of the action

$$S = \int \left\{ P\dot{R} + i[(\lambda\dot{\bar{\lambda}}) + (\bar{\lambda}\dot{\lambda})] - H_c \right\} dt, \quad (15)$$

where P is the momentum canonically conjugate to $R(t)$, and the canonical Hamiltonian H_c of the system has the following structure

$$H_c = N(t)H_0 + i\psi^a(t)\bar{Q}_a + i\bar{\psi}_a(t)Q^a - V^i(t)\mathcal{F}_i, \quad (16)$$

with

$$H_0 = -\frac{\mathfrak{a}^2 P^2}{12R} - \frac{3kR}{\mathfrak{a}^2} - \frac{\sqrt{k}}{R}(\bar{\lambda}\lambda) + \frac{\mathfrak{a}^2}{8R^3}(\bar{\lambda}\lambda)(\lambda\lambda) \quad (17)$$

$$\bar{Q}_a = \left(\frac{\mathfrak{a}R^{-1/2}P}{\sqrt{6}} + i\frac{\sqrt{k}\sqrt{6R}}{\mathfrak{a}} \right) \bar{\lambda}_a - 3i\mathfrak{a}(6R)^{-3/2}(\bar{\lambda}\lambda)\lambda_a, \quad (18)$$

$$Q^b = \left(\frac{\mathfrak{a}R^{-1/2}P}{\sqrt{6}} - i\frac{\sqrt{k}\sqrt{6R}}{\mathfrak{a}} \right) \lambda^b - 3i\mathfrak{a}(6R)^{-3/2}\bar{\lambda}^b(\lambda\lambda), \quad (19)$$

and

$$\mathcal{F}_i = (\lambda\sigma_i\bar{\lambda}), \quad (20)$$

where H_0 is the Hamiltonian of the system, Q^a and \bar{Q}_a are the supercharges, and \mathcal{F}_i are the SU(2) rotations. These formulae for the conserved supercharges complete the classical description of the desired $n = 4$ SUSY QM and now to quantize it we should analyze it's constraints. Following the standard procedure of quantization of the system with bosonic and fermionic degrees of freedom, we introduce the canonical Poisson brackets

$$\{R, P\} = 1, \quad \{\lambda^a, \pi_{(\lambda)_b}\} = -\delta_b^a, \quad \{\bar{\lambda}_a, \pi_{(\bar{\lambda})}^b\} = -\delta_a^b, \quad (21)$$

where $P, \pi_{(\lambda)}$ and $\pi_{(\bar{\lambda})}$ are the momenta conjugated to R, λ^a and $\bar{\lambda}_a$. From the explicit form of the momenta

$$P = -\frac{6R}{\mathfrak{a}^2 N} DR = -\frac{6R}{\mathfrak{a}^2 N} \left\{ \dot{R} - \frac{i\mathfrak{a}}{\sqrt{6R}} [(\bar{\psi}\lambda) - (\bar{\lambda}\psi)] \right\}, \quad (22)$$

$$\pi_{a(\lambda)} = i\bar{\lambda}_a, \quad \pi_{(\bar{\lambda})}^a = i\lambda^a, \quad (23)$$

one can conclude, that the system possesses the secon-class fermionic constraints

$$\Pi_{(\lambda)_a} = \pi_{(\lambda)_a} - i\bar{\lambda}_a, \quad \Pi_{(\bar{\lambda})}^a = \pi_{(\bar{\lambda})}^a - i\bar{\lambda}^a, \quad (24)$$

since

$$\{\Pi_{(\bar{\lambda})}^a, \Pi_{(\lambda)_b}\} = 2i\delta_b^a. \quad (25)$$

Therefore, the quantization has to be done using the Dirac brackets defined for any two functions F and G

$$\{F, G\}^* = \{F, G\} - \{F, \Pi_a\} \frac{1}{\{\Pi_a, \Pi_b\}} \{\Pi_b, G\}. \quad (26)$$

As a result, we obtain the following Dirac brackets for the canonical variables

$$\{R, P\}^* = 1, \quad \{\lambda^a, \bar{\lambda}_b\}^* = \frac{i}{2} \delta_b^a. \quad (27)$$

The supercharges and the Hamiltonian form the following $n = 4$ SUSY QM algebra with respect to the introduced Dirac brackets

$$\{\bar{Q}_a, Q^b\}^* = -i\delta_a^b H_0, \quad \{\mathcal{F}_i, \mathcal{F}_j\}^* = \epsilon_{ijk} \mathcal{F}_k, \quad (28)$$

$$\{\mathcal{F}_i, \bar{Q}_a\}^* = \frac{i}{2} (\sigma_i)_a^c \bar{Q}_c, \quad \{\mathcal{F}_i, Q^a\}^* = -\frac{i}{2} (\sigma_i)_c^a Q^c. \quad (29)$$

On the quantum level we replace the Dirac brackets by (anti)commutators using the rule

$$i\{\cdot, \cdot\}^* = [\cdot, \cdot]. \quad (30)$$

One obtains the non-zero commutation relations for the Dirac brackets

$$[R, P] = i, \quad \{\lambda^a, \bar{\lambda}_b\} = -\frac{1}{2} \delta_b^a. \quad (31)$$

In the quantum theory the first-class constraints (17-20) associated with the invariance of the action (14) become conditions on the wave function Ψ of the Universe. Therefore, any physically allowed states must obey the quantum constraints

$$H_0 \Psi = 0, \quad Q^a \Psi = 0, \quad \bar{Q}_a \Psi = 0, \quad \mathcal{F}_i \Psi = 0. \quad (32)$$

The quantum generators H_0, Q^a, \bar{Q}_a and \mathcal{F}_i form a closed superalgebra of the $n = 4$ supersymmetric quantum mechanics

$$\begin{aligned} \{\bar{Q}_a, Q^b\} &= H_0 \delta_a^b, & [\mathcal{F}_i, \mathcal{F}_j] &= i\epsilon_{ijk} \mathcal{F}_k, & [\mathcal{F}_i, \bar{Q}_a] &= -\frac{1}{2} (\sigma_i)_a^b \bar{Q}_b, \\ [\mathcal{F}_i, Q^a] &= \frac{1}{2} (\sigma_i)_b^a Q^b. \end{aligned} \quad (33)$$

In the usual quantization the even canonical variables are replaced by operators $R \rightarrow R, P \rightarrow -i\frac{\partial}{\partial R}$ and the odd variables λ and $\bar{\lambda}$ after quantization become anticommuting operators in accordance with (31). In order to obtain the quantum expression for the Hamiltonian H_0 and for the supercharges Q^a and \bar{Q}_a we must solve the operator ordering ambiguity. Such ambiguities always arise when, as in our case, the operator expression contains the product of non-commuting operators R and P [13]. Technically it means the following: for the quantum supercharges we take the same order as for the operator (18,19). These calculations are performed doing the integration with the measure $R^{1/2} dR$. With this measure the conjugate

momentum $P = -i\frac{\partial}{\partial R}$ is non-Hermitian with $P^\dagger = R^{-1/2}PR^{1/2}$. However, the combination $(R^{-1/2}P)^\dagger = P^\dagger R^{-1/2} = R^{-1/2}P$ is Hermitian and $(R^{-1/2}PR^{-1/2}P)^\dagger = R^{-1/2}PR^{-1/2}P$ is also Hermitian. So the anticommutation relation $\{Q^a, Q_b^\dagger\} = \delta_b^a H_0$ fix all additional terms and define the quantum Hamiltonian, but in this case the operational expression $-\frac{\mathfrak{A}^2}{12}(R^{-1/2}PR^{-1/2}P)$ corresponds to the energy of the scale factor R in the Hamiltonian (33). As we can see from the classical Hamiltonian (33), the energy of the scale factor is negative. This is due to the fact that the particle-like fluctuations do not correspond to the scale factor $R(t)$.

III. CONCLUSIONS

On the basis of local $n = 4$ supersymmetry the superfield action for the FRW cosmological model was formulated. Due to the quantum supersymmetric algebra (33), the Wheeler-DeWitt equation, which is of the second-order, can be replaced by the four supercharge operator equations constituting its supersymmetric “square root”.

It would be very interesting to generalize the proposed $n = 4$ superfield model to the all Bianchi-type models [8], as well as, to consider interaction with matter fields and analyze the spontaneous breaking of the $n = 4$ local supersymmetry. We hope that for this more general supersymmetric cosmological models than [11], we can find a normalizable wavefunction. The details of this searches will be given elsewhere.

Acknowledgments. We are grateful to J. Socorro and O. Obregón for their interest in the work and useful comments. This research was supported in part by CONACyT under the grants 2845-54E and 28454E. Work of A.P. was supported in part by the RFBR under the grant 99-02-18417 and the joint grant RFBR-DFG 99-02-04022, and work of M.T. by RFBR 99-02-18417 and RFBR 01-02-06531. One of us (J.J.R.) would like to thank CONACyT for support under Estancias Posdoctorales en el Extranjero.

-
- [1] E. Witten, Nucl. Phys. **B185**, 513 (1981), E. Witten, Nucl. Phys. **B202**, 253 (1982).
 - [2] E.A. Ivanov, S.O. Krivonos and A.I. Pashnev, Class. Quantum Grav. **8**, 19 (199), V.P. Berezhovoj and A.I. Pashnev, Preprint KFTI 91-20, Kharkov, (1991).
 - [3] A.I. Pashnev and D.P. Sorokin, Phys. Lett.**B253**, 301 (1991).
 - [4] E.E. Donets, A. Pashnev, J.J. Rosales and M. Tsulaia, Phys. Rev. **D61**, 043512 (2000).
 - [5] R. Britto-Pacumio, J. Michelson, A. Strominger, A. Volovich, *Lectures on Superconformal Mechanics and Multi-Black Hole Moduli Spaces*, hep-th/9911066.
 - [6] G. Papadopoulos, Class. Quantum Grav. **17**, 3715 (2000).
 - [7] O. Obregón, J.J. Rosales and V.I. Tkach, Phys. Rev. **D53**, 1750 (1996).
 - [8] V.I. Tkach, J.J. Rosales and O. Obregón, Class. Quantum Grav. **13**, 2349 (1996).
 - [9] A. Macías, O. Obregón and M.P.Jr. Ryan, Class. Quantum Grav. **4**, 1477.
 - [10] P.D. D’Eath, *Supersymmetric Quantum Cosmology* (Cambridge: Cambridge University Press, 1999).
 - [11] O. Obregón, J.J. Rosales, J. Socorro and V.I. Tkach, Class. Quantum Grav. **16**, 2861 (1999).

- [12] E.E. Donets, A. Pashnev, V.O. Rivelles, D. Sorokin and M. Tsulaia, Phys. Lett. **B 484**, 337 (2000).
- [13] V. De Alfaro, S. Fubini, G. Furlan and M. Roncadelli, Nucl. Phys. **B 296**, 402 (1988); V.P. Berezovoj and A.I. Pashnev, Class. Quantum Grav. **8**, 2141 (1991).